COMPUTATION OF JOINT EXTENSIBILITY REQUIREMENTS

This technical release presents the procedure and working tools required for the computation of the joint extensibility that may be required in a drop inlet barrel constructed of articulated segments which are essentially free to move with the adjacent parts of the embankment or earth foundation. The discussion and procedures that are established for determining the depth "d" in which foundation compression occurs, the average foundation shear strength "s" as used to compute foundation stress ratio, and the corresponding foundation settlement "δ" relate only to the computation of the required joint extensibility of conduits on yielding foundations. The foundation is considered as a body and conduit cuts or pads are not considered as influencing the total foundation deformations. These procedures do not necessarily apply to situations involving a determination of total foundation settlement.

An explanation of the strains produced at or near the interface of an earth dam embankment and its compressible foundation is contained in two reports. They are (1) "Report on Investigation of Deformations in Foundations of Earth Embankments Containing Concrete Pressure Pipe Conduits" by Moran, Proctor, Mueser and Rutledge, Consulting Engineers, dated September 1960 and (2) "Report on Study of Movements of Articulated Conduits Under Earth Dams on Compressible Foundations" by Mueser, Rutledge, Wentworth and Johnston, Consulting Engineers, dated June 1968. These reports provide the basic data and procedure which are used herein to estimate joint extensibility requirements.

The depth of the compressible foundation, d, will be obvious in some cases but in others it may be obscure until consolidation computations based on proper evaluation of foundation conditions and laboratory tests indicate the depth below which consolidation may be neglected. When the compressive unit strain in feet per foot in any stratum under the center of the embankment and at a depth of about 0.25H or more becomes less than 10 percent of the compressive unit strain of the strata above, and strata with a higher compressive unit strain do not exist below the stratum in question, it may be assumed that the depth of the compressible foundation has been attained. Obviously judgment is required in estimating d and the consolidation potential of the foundation. Relatively large consolidation can be expected on loessial soils which have not been preloaded, medium stiff residual soils or special fine grained material such as glacial lake deposits whereas relatively low or insignificant consolidation should be anticipated from ordinary SCS dams on glacial till, stream terraces, or alluvial coarse sands and gravels.
It is important that the maximum settlement, \( \delta \), be estimated with reasonable accuracy. A quotation from page 39 of the 1968 report reads as follows, "It is recommended that the settlement analysis concentrate attention on the evaluation of the probable preconsolidation condition determined from consolidation tests, but also utilizing geological evidence and data from undrained shear tests. If it can be estimated that the foundation is overconsolidated, a nominal value of recompression index should be used in computing settlements, rather than to estimate \( \Delta e \) directly from the e - log p curve." The straight-line semi-log recompression index ordinarily may be estimated within the range from 0.04 for lightly over-consolidated plastic clays to 0.015 for heavily over-consolidated hard or dense mixtures of silt and clay with sand or gravel. The recompression index is a dimensionless parameter which equals the void-ratio decrement for one cycle of increase of effective stress.

The shear strength of the foundation, \( s \), must be estimated as realistically as possible. The shear strength in question is an average strength of the weakest stratum in the foundation at or near the interface with the embankment. Mr. Homer Cappleman estimated in a paper titled "Movements in Pipe Conduits Under Earth Dams" published in Journal, Soil Mechanics and Foundations Division, ASCE, November 1967, that foundation strata at a depth of more than 0.1B could be ignored in this determination.

If the size of the earth dam justifies fairly extensive testing of undisturbed samples of foundation soils, the shear strength may be estimated as follows. The probable average shear strength at the end of construction under a small earth dam is obtained from a consolidated-undrained triaxial test in which the chamber pressure is set equal to about two-thirds the average effective stress, \( \bar{p} \), at the depth in question.

The average effective stress, \( \bar{p} \), at the completion of the embankment is

\[
\bar{p} = \frac{H}{2} \gamma_s + y \gamma' f
\]  

(1)

Where

- \( \bar{p} \) = average effective stress on stratum in lb./ft.\(^2\)
- \( y \) = depth into the foundation from the embankment-foundation interface to the stratum in question in feet.
- \( \gamma' f \) = submerged weight of foundation material in lb./ft.\(^2\)

If detailed strength testing is not justified, the shear strengths may be estimated from preconsolidation data in the following manner. The
preconsolidation stress, $P$, has a very significant effect on shear strength and may be used to determine the average shear strength for silts, clays and other fine grained soils with a high percentage of silt or clay or both. For soils in which the preconsolidation stress exceeds the load to be applied by the embankment, the shear strength, $s$, should be taken as $0.3P$. For underconsolidated soils where the preconsolidated stress is less than applied load, the shear strength should be taken as $0.3$ of the effective stress at the stratum in question and under the midheight of the earth dam embankment (the average effective stress) multiplied by a factor $C$ which ranges between 0.75 and 0.9.

$$s = 0.3P < C < 0.75 \text{ or } 0.9$$ (2)

The factor $C$ is estimated between 0.75 and 0.9 from a consideration of the depth of the stratum and the strength of the material between it and the interface. If the stratum under consideration is just below the interface the factor $C$ should be taken as 0.75 where as if it is a depth $y$ which approaches $0.1B$ and the strength of foundation strata above are significantly greater, then $C$ should be taken as 0.9.

Consolidation tests of undisturbed samples from the various foundation strata will indicate the preconsolidation stress. Geologic history of the site is valuable in predicting the possibility of preconsolidation and its order of magnitude as a check against the consolidation test data. Recent alluviums may indicate moderate preconsolidation to a depth of several feet due to dessication, having strata below with little preconsolidation and low shear strength that were deposited in water and have had little opportunity to dry out.

Compute joint extensibility requirements in conformance with the following procedure.

**Step 1.** Compute the following ratios, $B \div d$, $B \div H$, $\delta \div d$, $(2pd) \div sB$ and $p = H \div d$.

**Step 2.** From ES-146 read, $R_1$, the theoretical ratio of maximum unit horizontal strain to average unit vertical strain, $\delta \div d$.

**Step 3.** Compute $R_2$, a factor which corrects for the effect of the foundation stress ratio, $\frac{2pd}{sB}$, on the theoretical ratio $R_1$.

$$R_2 = \frac{2pd}{sB} + 0.10$$ (3)

**Step 4.** Compute $e_{hz}$, the maximum unit horizontal strain.

$$e_{hz} = R_1 \cdot R_2 \cdot \frac{\delta}{d}$$ (4)
Step 5. Compute $g_s$, the maximum probable joint opening due to foundation and embankment strain

$$g_s = e_{u_s} \cdot 12 \cdot L$$  \hspace{1cm} (5)

where $L$ is the length of a section of conduit in feet. It is assumed that the articulated conduit under the major part of embankment is made up of sections of equal length, $L$.

![Fig. 1 Definition sketch](image)

Available evidence indicates that, as the conduit (barrel) settles, the induced rotation in the joints is not consistent but rather is quite irregular to the extent that in some cases the rotation is opposite to the anticipated direction. This situation probably is due to localized irregularities in the foundation, its consolidation potential, and the effect of anti-seep collars on differential settlement of the conduit.

Step 6. The probable joint opening due to joint rotation, $g_r$, in inches may be computed from the following equation which was derived from experimental data

$$g_r = \frac{2.5D_o \delta}{B}$$  \hspace{1cm} (6)

where $D_o$ = outside diameter or vertical height of conduit in inches.

Step 7. The required joint extensibility, $J$, in inches is given by the following equation

$$J = g_s + g_r + S$$  \hspace{1cm} (7)
where \( S \) is the safety margin in inches. The safety margin, \( S \), is the larger value given by equation (8) or the requirements of Engineering Memorandum-27.

\[
S = \frac{1}{2} \cdot \frac{2pd}{sB} + C_H + C_D \quad . \quad . \quad . \quad . \quad . \quad (8)
\]

where

\[
C_H = \frac{h - 100}{100} \text{ for } (H > 100)
\]

\[
= 0 \quad \text{for } (H \leq 100) \quad . \quad . \quad . \quad . \quad . \quad (9)
\]

\[
C_D = \frac{30 - D}{30} \text{ for } (D < 30)
\]

\[
= 0 \quad \text{for } (D \geq 30) \quad . \quad . \quad . \quad . \quad . \quad (10)
\]

The required joint length (EM-27) is equal to the required joint extensibility plus the maximum joint gap permitted when the pipe is installed.

Nomenclature Summary:

- \( B \) = equivalent base width of embankment in feet.
- \( C \) = coefficient (see equation 2)
- \( C_H \) = a part of the safety margin in inches (see equation 9)
- \( C_D \) = a part of the safety margin in inches (see equation 10)
- \( d \) = depth of the compressible foundation, i.e. that depth in the foundation below the interface, below which additional significant settlement does not occur, in feet.
- \( D \) = internal diameter or inside vertical height of conduit in inches
- \( D_o \) = maximum outside diameter or vertical height of conduit in inches
- \( g_s \) = maximum probable joint opening due to foundation and embankment strain in inches (see equation 5)
- \( g_r \) = probable joint opening due to joint rotation in inches (see equation 6)
- \( H \) = height of earth embankment in feet
- \( J \) = required joint extensibility in inches (see equation 7)
\( L = \) length of a monolithic section of conduit in feet

\( P = \) preconsolidation stress in pounds per square foot

\( p = H \gamma_s = \) maximum vertical pressure at the interface in pounds per square foot

\( \bar{p} = \) average effective stress on stratum at depth \( y \) in pounds per square foot

\( R_1 = \) theoretical ratio of maximum unit horizontal strain to average unit vertical strain, \( \delta \div d \)

\( R_2 = \) a correction factor for the effect of the foundation stress ratio on \( R_1 \) (see equation 3)

\( s = \) average consolidated undrained foundation shear strength at the condition of completion of the embankment in pounds per square foot

\( S = \) safety margin in inches (see equation 8)

\( y = \) depth into the foundation from the embankment-foundation interface to the stratum in question in feet

\( \varepsilon_{h,m} = \) maximum unit horizontal strain

\( \delta = \) maximum anticipated settlement of the foundation surface in the vicinity of the conduit in feet

\( \gamma_s = \) moist weight of the embankment as built in pounds per cubic foot

\( \gamma_f' = \) average submerged weight of foundation material above depth \( y \) in pounds per cubic foot
Example No. 1

Given:  
B = 280. ft.; H = 44. ft.; d = 12. ft.; δ = 0.85 ft.

γ₀ = 115. lb./ft.³; s = 1800. lb./ft.²; L = 16. ft.; D = 48. in.

D₀ = 54. in.; class (a) dam;

Find: Required joint extensibility

Procedure:

**Step 1.** Compute \( \frac{B}{d} = \frac{280}{12} = 23.3 \); \( \frac{B}{H} = \frac{280}{44} = 6.4 \); \( \frac{\delta}{d} = \frac{0.85}{12} = 0.071 \);

\[ p = H\gamma₀ = (44)(115) = 5060. \text{ lb.}/\text{ft.}²; \]

\[ \frac{2pd}{sB} = \frac{(2)(5060)(12)}{(1800)(280)} = 0.24 \]

**Step 2.** From ES-146 for \( \frac{B}{d} = 23.3 \) and \( \frac{B}{H} = 6.4 \) read \( R₁ = 0.123 \)

**Step 3.** \( R₂ = 0.24 + 0.10 = 0.34 \)

**Step 4.** \( \varepsilon_{hα} = (0.123)(0.34)(0.071) = 0.00297 \)

**Step 5.** \( gₛ = (0.00297)(12)(16) = 0.57 \text{ inch} \)

**Step 6.** \( gᵣ = \frac{(2.5)(54)(0.85)}{280} = 0.41 \)

**Step 7.** \( S = \left(\frac{1}{2}\right)(0.24) + 0 + 0 = 0.12 < 0.5 \text{ hence use } S = 0.5 \)

\[ J = 0.57 + 0.41 + 0.50 = 1.48 \text{ inches} \]
Example No. 2

Given: Cross section of earth dam embankment as shown; \( d = 26 \) ft.; 
\( \delta = 2.15 \) ft.; \( \gamma_a = 125 \) lb./ft.\(^2\); \( s = 1000 \) lb./ft.\(^2\); \( L = 10 \) ft.; 
\( D_o = 35 \) in.; \( D = 30 \) in. class (c) dam.

Find: Effective \( B \) and \( H \) and \( J \)

\( H = 41 \) ft. by inspection

\( B = \frac{2}{H} \) times cross-sectional area of dam = \( \frac{(2)(5333)}{41} = 260 \) ft.

Procedure:

Step 1. 
\[
\frac{B}{d} = \frac{260}{26} = 10; \quad \frac{B}{H} = \frac{260}{41} = 6.3; \quad \frac{\delta}{d} = \frac{2.15}{26} = 0.083
\]

\( p = H\gamma_a = (125)(41) = 5125 \) lb. per ft.\(^2\)

Foundation stress ratio, 
\[
\frac{2pd}{sB} = \frac{(2)(5125)(26)}{(1000)(260)} = 1.03
\]

Step 2. From ES-146 for \( \frac{B}{d} = 10 \) and \( \frac{B}{H} = 6.3 \) read \( R_1 = 0.213 \)

Step 3. 
\( R_o = 1.03 + 0.10 = 1.13 \)

Step 4. 
\( \epsilon_h = (R_1)(R_o) \left( \frac{\delta}{d} \right) = (0.213)(1.13)(0.083) = 0.020 \)

Step 5. 
\( g_s = (\epsilon_h)(L)(12) = (0.020)(10)(12) = 2.40 \) inches

Step 6. 
\( g_r = \frac{2.5D_o \delta}{B} \left( \frac{2.5}{2} \right) \left( \frac{35}{2} \right) = 0.72 \) inches

Step 7. 
\[
S = \frac{1}{2} \cdot \frac{2pd}{sB} + C_H + C_D
\]
\[
= \left( \frac{1}{2} \right)(1.03) + 0 + 0 = 0.52 > 0.5 \text{ use } S = 0.52
\]

Step 8. 
\( J = g_s + g_r + S = 2.40 + 0.72 + 0.52 = 3.64 \) inches
SOIL MECHANICS: Values of theoretical ratio of maximum unit horizontal strain to average unit vertical strain = \( R_1 \)

\[ \mu = 0.25 \]

\[ \text{Values of } \frac{B}{d} \]

Values of ratio \( R_1 = 0.14 \)

Values of ratio \( R_1 = 0.12 \)

Values of ratio \( R_1 = 0.10 \)

Values of ratio \( R_1 = 0.09 \)

Values of ratio \( R_1 = 0.08 \)

Values of ratio \( R_1 = 0.07 \)

Values of ratio \( R_1 = 0.06 \)

\[ \text{Maximum average unit vertical strain } e_{VM} = 8/d \]

Compressible foundation: \( \mu = 0.25 \)

REFERENCE: "Report on Investigation of Deformations in Foundations of Earth Embankments Containing Concrete Pressure Pipe Conduits" by Moran, Proctor, Mueser, and Rutledge.

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