SUBJECT: ENG - RIPRAP LINED PLUNGE POOL FOR CANTILEVER OUTLET

Purpose. To distribute Design Note No. 6 (Second Edition), "Riprap Lined Plunge Pool for Cantilever Outlet."

Effective Date. Effective when received.
Explanation. Design Note No. 6 was originally issued in 1969 and was based on research reported in 1967. This second edition was developed based on recently reported research by Fred W. Blaisdell of the Agricultural Research Service. The need for a revision of the original Design Note No. 6 has been identified by several reported situations of riprap being displaced from the plunge pool.

The Blaisdell developed mathematical model is in overall agreement with the experimental data. The purpose of this edition of the design note is to present Blaisdell's final design equations with modifications to facilitate construction and still meet the minimum design requirement.

Filing Instructions. Discard the 1969 edition of Design Note No. 6 and file this second edition with other design notes.

Distribution. The design note should be useful to professionals designing or reviewing the design of an energy dissipator at the downstream end of a conduit spillway. Initial distribution is shown on the reverse side. Additional copies may be obtained from Central Supply.


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Deputy Chief for Technology
Enclosure

Design Note No. 6 (Second Edition)*

Subject: Riprap Lined Plunge Pool for Cantilever Outlet

## INTRODUCTION

The energy in flow exiting from a spillway usually requires dissipation before being released to the outlet channel. For flow exiting from a conduit, when an open plunge pool is acceptable, an excavated ripraplined hole at the downstream end of the conduit can be an economical energy dissipator. However, the size of plunge pool, location relative to the conduit outlet, and size of riprap must be properly designed for the plunge pool to operate successfully. Successful operation consists of negligible kinetic energy in the outflow, no erosion or loss of the plunge pool foundation soil due to the turbulence in the process of energy dissipation, and no displacement of the riprap.

Design Note No. 6, originally issued in 1969, was based on research reported in 1967. This second edition was developed based on recently reported research (Blaisdell and Anderson 1984). The need for a revision was identified by several reported situations of riprap being displaced from the plunge pool.

Fred W. Blaisde11, Research Hydraulic Engineer, of the Agricultural Research Service conducted experiments at the Saint Anthony Falls Hydraulic Laboratory of the University of Minnesota in Minneapolis to evaluate the scour at cantilevered pipe spillway outlets. The Blaisdell developed mathematical model indicates an overall agreement with the experimental data. The final equations for the design of plunge pool energy dissipators for cantilevered pipe spillways were presented at the ASCE Hydraulic Division Conference in Coeur d'Alene, Idaho, August, 1984. The purpose of this edition of the design note is to present the final design equations with modifications to facilitate the evaluation of plunge pool shape, length, width, depth, position in relation to the outlet end of the conduit, and plunge pool volumes. Figures 1 and 2 illustrate the plunge pool layout dimensions.

## DISCHARGE PARAMETER

The plunge pool dimensions were developed using a discharge parameter. The parameter is based on the design discharge, Q , pipe diameter, D , and combined with the acceleration of gravity, $g$, resulting in a

[^0] D.C.
dimensionless parameter of $\frac{Q}{\sqrt{\mathrm{gD}^{5}}}$. This is very convenient since $Q$ and $D$ are known prior to the plunge pool design.


Figure 1 -- Plunge pool definition sketch

## DISCHARGE JET TRAJECTORY

The plunge pool location is determined by the discharge jet trajectory. The location of the plunge pool centerline downstream from the discharge end of the pipe is dependent on the jet velocity and angle of impingement with the pool surface as well as the plunge pool depth.

The jet impingement velocity and angle of entry into the pool can be determined from the pipe exit slope, pipe discharge velocity, and height of pipe invert above the water surface. The height of pipe invert above the water surface, $Z_{p}$, should be measured from the tailwater elevation for the associated discharge used for the plunge pool design. The discharge should be the maximum prior to any secondary spillway flow. The pipe slope is $\frac{S}{\sqrt{1-S^{2}}}$, where $S$ is the sine of the angle whose tangent is the slope of the pipe. The discharge velocity, $\mathrm{V}_{\mathrm{o}}$, is computed based on the design discharge and the conduit cross-sectional area. The path of the free falling jet is a parabola between the pipe exit and tailwater surface where the jet enters the water with the impingement velocity, $\mathrm{V}_{\mathrm{p}}$, and the slope, tan $\alpha$. The horizontal distance, $X_{p}$, from the pipe exit to where the jet plunges into the tailwater with horizontal velocity, $\mathrm{V}_{\mathrm{h}}$, and vertical velocity, $\mathrm{V}_{\mathrm{v}}$, is given in Eq. 5; where

$$
\begin{aligned}
& v_{h}=v_{0} \cos \left(\sin ^{-1} s\right) \\
& v_{v}=\sqrt{\left(v_{0} s\right)^{2}+2 g\left[z_{p}+\frac{D}{2} \cos \left(\sin ^{-1} s\right)\right]} \\
& \tan \alpha=\frac{v_{v}}{v_{h}} \\
& v_{p}=\sqrt{v_{h}^{2}+v_{v}^{2}} \\
& X_{p}=\frac{v_{h}}{g}\left(v_{v}-v_{o} s\right)
\end{aligned}
$$

Eq. 1

Eq. 2

Eq. 3

Eq. 4

Eq. 5

PLUNGE POOL DEPTH

The depth of erosion created by the discharging jet can be controlled by the bed material size. The bed material is represented by its mean grain size, $d_{50}$, the size of which 50 percent by weight is finer in diameter. The research tests were run on noncohesive materials. Therefore, this design procedure is appropriate for soil and rock bed material that perform as single grain material in resisting erosion. The $d_{50}$ size for riprap lining material may be varied to adjust the erosion depth. The plunge pool depth is computed using a densimetric Froude number, $F_{d}$, as follows:

$$
F_{d}=\frac{V_{p}}{\sqrt{g d_{50}\left(\rho_{s}-\rho\right) / \rho}}
$$

Eq. 6
where: $\quad \rho_{s}=$ Bed material or riprap particle density $\rho=$ Water density

For $\frac{Z_{p}}{D}<1$, the maximum eroded depth is computed by the equation

$$
Z_{m}=7.5 \mathrm{D}\left[1-\mathrm{e}^{-0.6\left(\mathrm{~F}_{\mathrm{d}}-2\right)}\right] \quad \text { Eq. 7a }
$$

For $\frac{Z_{p}}{D}>1$, the maximum eroded depth is computed by the equation

$$
Z_{m}=10.5 D\left[1-e^{-0.35\left(F_{d}-2\right)}\right] \quad \text { Eq. 7b }
$$

The effect of a horizontal ledge or a nonerodible layer on the shape of the plunge pool above the layer was tested and found to be a minimal.

The dimensions of the plunge pool are functions of $\frac{Z_{m}}{D}$. When the plunge pool dimensions are based on the value of $z_{m}$, the designed contours above the ledge conform to the plunge pool shape. Therefore, it is acceptable to size and construct the plunge pool to 0.8 of the computed maximum depth, $Z_{m}$. However, the full value of the computed $Z_{m}$, as determined by equation 7 a or 7 b , must be used in subsequent equations 9, 10 , and 11.

The $d_{50}$ bed material size must be checked to assure that it is adequate to control shallow beach type erosion at the top edge of the plunge pool. High flow rates during research testing caused flow to circulate upstream along both sides of the plunge pool. When these circulating flows exceeded the bed material's critical tractive stress, beach erosion at the top edge of the plunge pool occurred. The check for adequate bed material size up to the tailwater elevation is by equation 8 . The $\mathrm{d}_{50}$ size is adequate and beach erosion will not occur if

$$
\begin{equation*}
\frac{Q}{\sqrt{g D^{5}}} \leqslant\left[1.0+25 \frac{d_{50}}{D}\right] \tag{Eq. 8}
\end{equation*}
$$

If the bed material $d_{50}$ is not large enough, protection will need to be added. In the case of riprap, a larger particle gradation will be required.

LOCATION OF PLUNGE POOL
The horizontal distance, $X_{m}$, from the pipe exit to the center of the plunge pool, i.e., where maximum scour depth occurs is

$$
X_{m}=\left[X_{p}+\frac{Z_{m}}{\tan \alpha}\right] 1.15 e^{-0.15\left[Q /\left(g D^{5}\right)^{1 / 2}\right]}
$$

Eq. 9

> DIMENSION OF PLUNGE POOL

The plunge pool natural shape is an ellipse with the greater length parallel to the pipe flow. The minimum size based on laboratory tests is the result of flow turbulence, boundary tractive stresses and submerged angle of repose of granular material. The test material $d_{50}$ sizes ranged from 0.5 to 8 mm . The minimum horizontal distance from the center of the plunge pool to the water surface contour at the upstream end of the pool is equal to $\mathrm{L}_{\mathrm{e}}$.

$$
\begin{aligned}
L_{e}=Z_{m}\left[\frac{3}{2}+\frac{1}{3} \frac{Q}{\sqrt{g D^{5}}}\right] & \text { Eq. } 10 \\
& (210-V I-D N-6, \text { Second Ed., January 1986) }
\end{aligned}
$$



Pigure 2 - Plunge Pool

Since the plunge pool shape is that of an ellipse, the distance from the center of the plunge pool to the water surface contour at the projected scour hole slope at the downstream end is also equal to the minimum $\mathrm{L}_{\mathrm{e}}$.

The minimum width of the plunge pool at the center of the pool is equal to $2 W_{e}$.

$$
W_{e}=Z_{m}\left[1.5+0.15 \frac{Q}{\sqrt{g D^{5}}}\right]
$$

Eq. 11

Once the minimum width, length, depth and the distance from the end of the pipe to the center of the plunge pool are determined for a given spillway layout and $d_{50}$ particle size, the final design shape and dimensions can be estabilshed. It is suggested that a comparable rectangular shape with length equal to $2 \mathrm{~L}_{e}$ and width equal to $2 \mathrm{~W}_{e}$ be specified thus facilitating construction and still meeting the minimum design requirement. The dimensions of the rectangular base at the bottom of the plunge pool, $0.8 \mathrm{Z}_{\mathrm{m}}$ below the water surface, are length, ${ }^{2} \mathrm{~L}_{\mathrm{r} 2}$, and width, $2 W_{r 2}$ where;

$$
\begin{align*}
2 \mathrm{~L}_{\mathrm{r} 2} & =0.4 \mathrm{~L}_{\mathrm{e}}  \tag{Eq. 12}\\
2 \mathrm{~W}_{\mathrm{r} 2} & =0.4 \mathrm{~W}_{\mathrm{e}}
\end{align*}
$$

It is recommended that the excavated side slopes of the plunge pool along the length, $z_{\ell}$ and along the width, $z_{\omega}$, be adjusted to acceptable grades for layout and riprap placement purposes, e.g., 3 horizontal to 1 vertical. The final length and width of the plunge pool at the water surface are $2 L_{r}$ and $2 W_{r}$, respectively. Where;

$$
\begin{align*}
& \mathrm{L}_{\mathrm{r}}=0.8 \mathrm{Z}_{\mathrm{m} \ell}^{\mathrm{z}}+\mathrm{L}_{\mathrm{r} 2}  \tag{Eq. 14}\\
& \mathrm{~W}_{\mathrm{r}}=0.8 \mathrm{Z}_{\mathrm{m} \omega} \mathrm{z}_{\omega}+\mathrm{W}_{\mathrm{r} 2} \tag{Eq. 15}
\end{align*}
$$

If $L_{r}$ is less than $X_{m}$, the water surface contour at the upstream end of the pool is downstream from the end of the conduit. Therefore, $\mathrm{L}_{\mathrm{r}}$ should be increased to equal to or greater than $X_{m}$.

## PLUNGE POOL VOLUMES

The volume, $V$, in cu. yds. of the plunge pool and lining materials, such as riprap and granular filter, can be determined as frustums of pyramids. For convenience, the appropriate equation is listed below

$$
V=\frac{1}{81}\left[A_{1}+A_{2}+\sqrt{A_{1} A_{2}}\right] z
$$

where $\quad A_{1}$ is the plan rectangular area of the plunge pool at the invert elevation of the outlet channel, $\mathrm{ft}^{2}$
$A_{2}$ is the plan rectangular area at the bottom of the plunge pool at a distance $Z$ below the invert elevation of the outlet channel, $\mathrm{ft}^{2}$

Z is either equal to $0.8 Z_{m}-Z_{d}, 0.8 Z_{m}-Z_{d}+a_{1}$, or $0.8 Z_{m}-Z_{d}+a_{2}$ below the invert elevation of the outlet channel, ft.
$a_{1}$ is the thickness of the riprap lining, ft. $a_{2}$ is the thickness of the riprap lining and granular filter material, ft.
$Z_{d}$ is the water depth above the invert elevation of the outlet channel, ft.

The volumes of riprap and filter material above the invert elevation of the outlet channel depend on the site topography.

PROCEDURE
The step procedure given below is in a form that can easily be programmed on either programmable calculators or microcomputers.

1. Compute $\frac{Q}{\sqrt{g D^{5}}}$
2. Compute $V_{0}=\frac{4 Q}{\pi D^{2}}$
3. Compute $V_{h}=V_{0} \cos \left(\sin ^{-1} \mathrm{~S}\right)$

Eq. 1
$V_{v}=\sqrt{\left(V_{0} S\right)^{2}+2 g\left[z_{p}+\frac{D}{2} \cos \left(\sin ^{-1} S\right)\right]} \quad$ Eq. 2
$\tan \alpha=\frac{V_{V}}{V_{h}}$
Eq. 3

$$
v_{p}=\sqrt{v_{h}^{2}+v_{v}^{2}}
$$

Eq. 4

$$
X_{p}=\frac{V_{h}}{g}\left(V_{v}-V_{0} S\right)
$$

Eq. 5
4. Compute $F_{d}=\frac{V_{p}}{\sqrt{g d_{50}\left(\rho_{s}-\rho\right) / \rho}}$

Eq. 6
5. Compute $\frac{Z_{p}}{D}$; if $\leqslant 1$, $G o$ to step $6 a$; if $>1$, Go to step $6 b$

6a. Compute $Z_{m}=7.5 \mathrm{D}\left[1-e^{-0.6\left(F_{d}-2\right)}\right]$; Go to step 7 Eq. 7a

6b. Compute $Z_{m}=10.5 \mathrm{D}\left[1-\mathrm{e}^{-0.35\left(\mathrm{~F}_{\mathrm{d}}-2\right)}\right]$
Eq. 7b
7. Compute $1.0+25 \frac{\mathrm{~d}_{50}}{\mathrm{D}}$

Eq. 8
8. If $\frac{Q}{\sqrt{g D^{5}}} \leqslant 1.0+25 \frac{d_{50}}{D}$, then go to step 9 ; otherwise, make
design adjustments to increase $d_{50}$ and return to step 4.
9. Compute $X_{m}=\left[X_{p}+\frac{Z_{m}}{\tan \alpha}\right] 1.15 e^{-0.15\left[Q /\left(g D^{5}\right)^{1 / 2}\right]} \quad$ Eq. 9
10. Compute $\mathrm{L}_{\mathrm{e}}=\mathrm{Z}_{\mathrm{m}}\left[\frac{3}{2}+\frac{1}{3} \frac{\mathrm{Q}}{\sqrt{\mathrm{gD}^{5}}}\right]$

$$
W_{e}=z_{m}\left[1.5+0.15 \frac{Q}{\sqrt{g D^{5}}}\right]
$$

11. Determine $A_{2}$, plan rectangular area of the plunge pool bottom at $0.8 \mathrm{Z}_{\mathrm{m}}$ below the water surface
$\mathrm{L}_{\mathrm{r} 2}=0.2 \mathrm{~L}_{\mathrm{e}}$
$\mathrm{W}_{\mathrm{r} 2}=0.2 \mathrm{~W}_{\mathrm{e}}$
$\mathrm{A}_{2}=4 \mathrm{~L}_{\mathrm{r} 2} \mathrm{~W}_{\mathrm{r} 2}$
12. Check the side slopes of the plunge pool and adjust, if necessary to acceptable grades, $z_{\ell}$ and $z_{\omega}$. The final length and width of the plunge pool at the water surface are $2 \mathrm{~L}_{\mathrm{r}}$ and $2 \mathrm{~W}_{r}$, respectively.
$\mathrm{L}_{\mathrm{r}}=0.8 \mathrm{Z}_{\mathrm{m}} \quad \mathrm{z}_{\ell}+\mathrm{L}_{\mathrm{r} 2}$
$W_{r}=0.8 \mathrm{Z}_{\mathrm{m}} \quad \mathrm{z}_{\omega}+\mathrm{W}_{\mathrm{r} 2}$
13. If $L_{r}<X_{m}$, increase side slope, $z_{\ell}$, so that $L_{r} \geqslant X_{m}$
14. Determine $A_{1}$, plan rectangular area of the plunge pool at the invert elevation of the outlet channel

$$
A_{1}=4\left(L_{r}-z_{\ell} Z_{d}\right)\left(W_{r}-z_{\omega} Z_{d}\right)
$$

15. Plunge Pool Volume:

The Volume between a horizontal plane at the invert elevation of the outlet channel and the exposed riprap surface is $\mathrm{V}_{\mathrm{ao}}$.

$$
v_{a o}=\frac{1}{81}\left[A_{1}+A_{2}+\sqrt{A_{1} A_{2}}\right]\left[0.8 Z_{m}-Z_{d}\right], \text { cu. yds. }
$$

The volume between a horizontal plane at the invert elevation of the outlet channel and a surface at a thickness, $a_{1}$, below the exposed riprap surface is $\mathrm{V}_{\mathrm{al}}$.

$$
V_{a l}=\frac{1}{81}\left[A_{1 a l}+A_{2 a l}+\sqrt{A_{1 a l} A_{2 a 1}}\right]\left[0.8 z_{m}-Z_{d}+a_{1}\right], \text { cu. yds. }
$$

where $A_{1 a l}=4\left[L_{r}-z_{\ell} Z_{d}+a_{1} \sqrt{1+z_{\ell}^{2}}\right]\left[W_{r}-z_{\omega} Z_{d}+a_{1} \sqrt{1+z_{\omega}^{2}}\right]$
and

$$
A_{2 a 1}=4\left[L_{r 2}+a_{1}\left(\sqrt{1+z_{\ell}^{2}}-z_{\ell}\right)\right]\left[W_{r 2}+a_{1}\left(\sqrt{1+z_{\omega}^{2}}-z_{\omega}\right)\right]
$$

The volume of riprap at thickness, $a_{1}$, below a horizontal plane at the invert elevation of the outlet channel, exclusive of the volume of the riprap filter cap is $\mathrm{V}_{\mathrm{al}}-\mathrm{V}_{\mathrm{ao}}$, cu . yds.

The volume between a horizontal plane at the invert elevation of the outlet channel and a surface at a thickness, $a_{2}$, below the exposed riprap surface is $\mathrm{V}_{\mathrm{a} 2}$
$v_{a 2}=\frac{1}{81}\left[A_{1 a 2}+A_{2 a 2}+\sqrt{A_{1 a 2} A_{2 a 2}}\right]\left[0.8 z_{m}-Z_{d}+a_{2}\right]$, cu. yds.
where $A_{l a 2}=4\left[L_{r}-z_{\ell} Z_{d}+a_{2} \sqrt{1+z_{\ell}^{2}}\right]\left[W_{r}-z_{\omega} z_{d}+a_{2} \sqrt{1+z_{\omega}^{2}}\right]$
and

$$
A_{2 a 2}=4\left[L_{r 2}+a_{2}\left(\sqrt{1+z_{\ell}^{2}}-z_{\ell}\right)\right]\left[W_{r 2}+a_{2}\left(\sqrt{1+z_{\omega}^{2}}-z_{\omega}\right)\right]
$$

The volume of filter material of thickness, $a_{2}-a_{1}$, below a horizontal plane at the invert elevation of the outlet channel, including the volume of the riprap filter cap, is equal to $\mathrm{v}_{\mathrm{a} 2}-\mathrm{v}_{\mathrm{al}}$, cu. yds.

## EXAMPLE

Given:
Invert elevation at outlet end of conduit $=102.5$
Invert elevation of outlet channel $=100.0$
Elevation of tailwater for maximum conduit discharge $=101.5$
$Q=147 \mathrm{cfs}, \quad D=2.5 \mathrm{ft} ., \quad \mathrm{S}=0$
Riprap size, $d_{50}=1.0 \mathrm{ft}, \rho_{\mathrm{s}}=2.64$
Thickness of filter material bed $=0.75 \mathrm{ft}$.

Determine:
I. Plunge pool position with respect to outlet end of conduit
II. Plunge pool depth, length, and width
III. Plunge pool volumes below the invert elevation of outlet channel

Solution:

1. $\frac{Q}{\sqrt{g D^{5}}}=\frac{147}{\sqrt{32.16(2.5)^{5}}}=2.62$
2. $V_{0}=\frac{4 Q}{\pi D^{2}}=\frac{4(147)}{3.14(2.5)^{2}}=30 \mathrm{ft} / \mathrm{sec}$
3. $V_{h}=V_{0} \cos \left(\sin ^{-1} \mathrm{~S}\right)=30 \mathrm{ft} / \mathrm{sec}$

$$
\begin{aligned}
V_{v} & =\left[\left(V_{o} S\right)^{2}+2 g\left(Z_{p}+\frac{D}{2} \cos \left(\sin ^{-1} \mathrm{~S}\right)\right)\right]^{1 / 2}=\left[0+64.32\left(1.0+\frac{2.5}{2}\right)\right]^{1 / 2} \\
& =12.0 \mathrm{ft} / \mathrm{sec}
\end{aligned}
$$

$\tan \alpha=\frac{V_{v}}{V_{h}}=\frac{12.0}{30}=0.40$
$V_{p}=\sqrt{v_{h}^{2}+v_{v}^{2}}=\sqrt{(30)^{2}+(12.0)^{2}}=32.3 \mathrm{ft} / \mathrm{sec}$
$X_{p}=\frac{V_{h}}{g}\left(V_{v}-V_{0} S\right)=\frac{30}{32.16}(12.0)=11.2 \mathrm{ft}$
4. $\quad F_{d}=\frac{V_{p}}{\sqrt{g d_{50}\left(\rho_{s}-\rho\right) / \rho}}=\frac{32.3}{\sqrt{32.16(1)(2.64-1) / 1}}=4.45$
5. $\frac{Z_{p}}{D}=\frac{1}{2.5}=0.4<1$, therefore use Equation 6a
6. $Z_{m}=7.5 D\left[1-e^{-0.6\left(F_{d}-2\right)}\right]=7.5(2.5)\left[1-e^{-1.47}\right]=14.4 \mathrm{ft}$
7. $1.0+25 \frac{d_{50}}{D}=1.0+25 \frac{1}{2.5}=11$
8. $\frac{Q}{\sqrt{g D^{5}}}<\left[1.0+25 \frac{d_{50}}{D}\right]$;
therefore riprap size is adequate to prevent significant shallow erosion enlargement at water surface elevation.
9. Plunge pool position from outlet end of pipe to center of pool, $X_{m}$

$$
\begin{aligned}
& X_{m}=\left[X_{p}+\frac{Z_{m}}{\tan \alpha}\right] 1.15 e^{-0.15\left(Q / \sqrt{g D^{5}}\right)} \\
& X_{m}=\left[11.2+\frac{14.4}{0.40}\right] 1.15 e^{-0.15(2.62)}=36.6 \mathrm{ft}
\end{aligned}
$$

10. Plunge Pool Dimensions
depth at center $=0.8 \mathrm{Z}_{\mathrm{m}}=0.8(14.4)=11.5 \mathrm{ft}$

$$
\begin{aligned}
& \mathrm{L}_{\mathrm{e}}=\mathrm{z}_{\mathrm{m}}\left[\frac{3}{2}+\frac{1}{3} \frac{Q}{\sqrt{\mathrm{gD}^{5}}}\right]=14.4\left[\frac{3}{2}+\frac{2.62}{3}\right]=34.2 \mathrm{ft} \\
& \mathrm{~W}_{\mathrm{e}}=\mathrm{z}_{\mathrm{m}}\left[1.5+0.15 \frac{Q}{\sqrt{\mathrm{gD}^{5}}}\right]=14.4[1.5+0.15(2.62)]=27.3 \mathrm{ft}
\end{aligned}
$$

11. $\mathrm{L}_{\mathrm{r} 2}=0.2 \mathrm{~L}_{\mathrm{e}}=0.2(34.2)=6.8 \mathrm{ft}$.
$W_{r 2}=0.2 W_{e}=0.2(27.3)=5.5 \mathrm{ft}$.
$A_{2}=4 L_{r 2} W_{r 2}=4(6.8)(5.5)=150 \mathrm{ft}^{2}$
12. $z_{\ell}=\frac{L_{e}-L_{r 2}}{0.8 z_{m}}=\frac{34.2-6.8}{11.5}=2.4$; Adjust $z_{\ell}$ to 3.0

$$
\therefore \mathrm{L}_{\mathrm{r}}=0.8 \mathrm{z}_{\mathrm{m}} \mathrm{z}_{\ell}+\mathrm{L}_{\mathrm{r} 2}=11.5(3.0)+6.8=41.3 \mathrm{ft}
$$

$$
z_{\omega}=\frac{W_{e}-W_{r 2}}{0.8 z_{m}}=\frac{27.3-5.5}{11.5}=1.9 ; \text { Ad just } z_{\omega} \text { to } 2.0
$$

$$
\therefore W_{r}=0.8 z_{\mathrm{m}} z_{\omega}+W_{r 2}=11.5(2.0)+5.5=28.5 \mathrm{ft}
$$

13. $L_{r}=41.3>X_{m}=36.6 \quad 0 . K$.
14. $A_{1}=4\left(L_{r}-Z_{\ell} Z_{d}\right)\left(W_{r}-z_{\omega} Z_{d}\right)=4[41.3-3(1.5)][28.5-2(1.5)]$
$=3754 \mathrm{ft}^{2}$
15. $V_{a o}=\frac{1}{81}\left[A_{1}+A_{2}+\sqrt{A_{1} A_{2}}\right]\left[0.8 Z_{m}-Z_{d}\right]$
$=\frac{1}{81}[3754+150+\sqrt{3754 \times 150}][11.5-1.5]=574 \mathrm{cu} . \mathrm{yds}$.

12

$$
\begin{aligned}
& A_{l a l}=4\left[L_{r}-z_{\ell} Z_{d}+a_{1} \sqrt{1+z_{\ell}^{2}}\right]\left[W_{r}-z_{\omega} Z_{d}+a_{1} \sqrt{1+z_{\omega}^{2}}\right. \\
& =4\left[41.3-3(1.5)+2.5 \sqrt{1+3^{2}}\right]\left[28.5-2(1.5)+2.5 \sqrt{1+2^{2}}\right] \\
& =5560 \mathrm{ft}^{2} \\
& A_{2 a 1}=4\left[L_{r 2}+a_{1}\left(\sqrt{1+z_{\ell}^{2}}-z_{\ell}\right)\right]\left[W_{r 2}+a_{1}\left(\sqrt{1+z_{\omega}^{2}}-z_{\omega}\right)\right] \\
& =4\left[6.8+2.5\left(\sqrt{1+3^{2}}-3\right)\right]\left[5.5+2.5\left(\sqrt{1+2^{2}}-2\right)\right]=176 \mathrm{ft}^{2} \\
& V_{a l}=\frac{1}{81}\left[A_{1 a l}+A_{2 a l}+\sqrt{A_{1 a l} A_{2 a l}}\right]\left[0.8 z_{m}-Z_{d}+a_{1}\right] \\
& =\frac{1}{81}[5560+176+\sqrt{5560 \times 176}][11.5-1.5+2.5]=1038 \mathrm{cu} . \mathrm{yds} . \\
& \text { Volume of riprap }=\mathrm{V}_{\mathrm{al}}-\mathrm{V}_{\mathrm{ao}}=1038-574=464 \mathrm{cu} \text {. yds. } \\
& A_{1 a 2}=4[41.3-4.5+3.25 \sqrt{10}][28.5-3+3.25 \sqrt{5}]=6170 \mathrm{ft}^{2} \\
& A_{2 a 2}=4[6.8+3.25(\sqrt{10}-3)][5.5+3.25(\sqrt{5}-2)]=184 \mathrm{ft}^{2} \\
& v_{a 2}=\frac{1}{81}[6170+184+\sqrt{6170 \times 184}][11.5-1.5+3.25]=1214 \mathrm{cu} . \text { yds. }
\end{aligned}
$$

Volume of filter $=\mathrm{V}_{\mathrm{a} 2}-\mathrm{V}_{\mathrm{a} 1}=1214-1038=176 \mathrm{cu} . \mathrm{yds}$.


## NOMENCLATURE

| $a_{1}$ | 三 | Thickness of riprap，ft |
| :---: | :---: | :---: |
| $a_{2}$ | 三 | Thickness of riprap and filter material，ft |
| $\mathbf{A}_{1}$ | 三 | Plan rectangular area of the plunge pool at the invert elevation of the outlet channel， $\mathrm{ft}^{2}$ |
| $\mathbf{A}_{2}$ | $\equiv$ | Plan rectangular area at the bottom of the plunge pool at a distance $Z$ below the invert elevation of the outlet channel， $\mathrm{ft}^{2}$ ． |
| $\mathrm{d}_{50}$ | ミ | Size of rock in riprap of which 50 percent by weight is finer，ft |
| D | $\equiv$ | Cantilever outlet pipe diameter，ft |
| e | ミ | Base of natural logarithms |
| $\mathrm{F}_{\mathbf{d}}$ | 三 | Densimetric Froude number |
| $g$ | 三 | Acceleration of gravity，ft／sec ${ }^{2}$ |
| $\mathrm{L}_{\mathrm{e}}$ | $\equiv$ | Minimum horizontal distance from the center of the pool to the water surface contour at the upstream or down－ stream end of an elliptical－shape plunge pool，ft |
| $L_{\mathbf{r}}$ | $\equiv$ | Adjusted horizontal distance from the center of the pool to the water surface contour at the upstream or downstream end of the rectangular－shape plunge pool，ft |
| $L_{r 2}$ | $\equiv$ | One－half＇the length of the bottom of a rectangular－shape plunge pool，ft |
| Q | $\equiv$ | Design discharge，cfs |
| S | $\equiv$ | Sine of the angle whose tangent is the slope of the pipe |
| $\mathrm{V}_{\text {ao }}$ | $\equiv$ | Volume of the plunge pool between the invert elevation of the outlet channel and the exposed riprap surface，cu．yds． |
| $\mathrm{V}_{\text {al }}$ | $\equiv$ | Volume of the plunge pool between the invert elevation of the outlet channel and a surface at a thickness，$a_{1}$ ， below the exposed riprap surface，cu．yds． |
| $\mathrm{V}_{\mathrm{a} 2}$ | $\equiv$ | Volume of the plunge pool between the invert elevation of the outlet channel and a surface at a thickness，$a_{2}$ ． below the exposed riprap surface，cu．yds． |
|  |  | （210－VI－DN－6，Second Ed．，January 1986） |


| $\mathrm{V}_{\mathrm{h}}$ | ミ | Horizontal component of the jet impingement velocity， $V_{p}$ ， $\mathrm{ft} / \mathrm{sec}$ |
| :---: | :---: | :---: |
| $\mathrm{v}_{0}$ | 三 | Velocity in the pipe corresponding to the design discharge， Q，ft／sec |
| $\mathrm{V}_{\mathrm{p}}$ | $\equiv$ | Velocity where the jet plunges into the water surface， ft／sec |
| $\mathrm{v}_{\mathrm{v}}$ | $\equiv$ | Vertical component of the jet impingement velocity，$V_{p}$ ， $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{W}_{\mathrm{e}}$ | $\equiv$ | One－half the minimum width at the center of the elliptical－ shape plunge pool at the water surface elevation，ft |
| $W_{r}$ | 三 | One－half the adjusted width at the center of the rectangular－ shape plunge pool at the water surface elevation，ft |
| $\mathrm{W}_{\mathrm{r} 2}$ | ミ | One－half the width of the bottom of a rectangular plunge pool，ft |
| $\mathrm{X}_{\text {m }}$ | 三 | Horizontal distance from the pipe exit to the center of the plunge pool，ft |
| $\mathrm{X}_{\mathrm{p}}$ | ミ | Horizontal distance from the pipe exit to the center of the jet plunging into the water surface，ft |
| ${ }^{2} \ell$ | 三 | Side slope ratio of the upstream or downstream slope of the rectangular－shape plunge pool |
| $2_{\omega}$ | ミ | Side slope ratio of the side slopes of the rectangular－ shape plunge pool |
| $\mathrm{Z}_{\mathrm{d}}$ | 三 | Water depth above the invert elevation of the outlet channel，ft |
| $\mathrm{Z}_{\mathrm{m}}$ | 三 | Maximum computed depth of the plunge pool，ft |
| $\mathrm{Z}_{\mathrm{p}}$ | 三 | Vertical distance from the tailwater surface to the cantilever pipe invert，ft |
| $p$ | $\equiv$ | Water density |
| ${ }^{\text {s }}$ | 三 | Bed material or riprap particle density |
| $\alpha$ | 三 | Jet impingement angle where the jet plunges into the water surface |

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